Magnetic-field–induced Luttinger liquid

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It is shown that a strong magnetic field applied to a bulk metal induces a Luttinger-liquid phase. This phase is characterized by the zero-bias anomaly in tunneling: the tunneling conductance scales as a power-law of voltage or temperature. The tunneling exponent increases with the magnetic field as $B \ln B$. The zero-bias anomaly is most pronounced for tunneling with the field applied perpendicular to the plane of the tunneling junction.

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A strong magnetic field applied to a bulk metal tends to reduce the effective dimensionality of charge carriers from 3D to 1D. This feature is most pronounced in the ultra-quantum limit, when only the lowest Landau level remains populated. The reduction of the effective dimensionality in a system of interacting particles is expected to result in a number of unusual phases, which are one-dimensional in nature, such as spin-and charge-density waves (CDW)[1–3], Wigner crystal[4], excitonic insulator (EI)[5], re-entrant superconductor[6], etc. A field-induced CDW is believed to have been observed in graphite (see, e.g., Ref. [7] for an extensive bibliography and discussion). There have been also earlier reports of tentative field-induced EI transitions in $Bi_{1-x}Sb_x$[8] and Bi[9].

In this paper, we focus on another field-induced state, which is not related to any instability, but evolves adiabatically from the conventional three-dimensional (3D) Fermi-liquid as the magnetic field increases. This state is a Luttinger Liquid (LL), whose existence can be anticipated from the following simplified picture. In a strong magnetic field, electron trajectories are helices spiraling around the field lines. A bundle of such trajectories with a common center of orbit can be viewed as a 1D conductor ("wire") with the Fermi velocity v_F^B determined from the condition of the fixed carrier density n in the bulk,

$$v_F^B = 2\pi^2 n \ell_B^2 / m_{||}. \tag{1}$$

Here $\ell_B = (eB)^{-1/2}$ is the magnetic length, and $m_{||}$ is the effective mass along the field; we assume also full spin polarization of electrons, and use the units with $\hbar = c = 1$. In the presence of electron-electron interactions, each "wire", considered separately, is in the LL-state. Interactions with small-momentum transfers among electrons on different "wires" do not change the LL-nature of a single-wire state[10].

The quantity of primary interest of this paper is the experimentally measurable density of states (DOS) at the sample boundary. We calculate this quantity first by treating perturbatively the interaction between electrons moving in 3D space in the presence of a quantizing magnetic field. We demonstrate that the boundary DOS exhibits a characteristic for a Luttinger liquid[11] power-law anomaly at the Fermi level:

$$\nu(\varepsilon) \propto |\varepsilon - \varepsilon_F^B|^{\alpha_s}, \tag{2}$$

where $\varepsilon_F^B = m_{||}(v_F^B)^2/2$ is the Fermi energy of the 1D motion along the magnetic field.

We establish the correspondence between a 3D electron plasma in a quantizing magnetic field and a Luttinger liquid by considering the electron-electron interaction in the basis of coherent states (CS) on the von Neumann lattice[12]. In this basis, a system of 3D electrons in the ultra-quantum limit is equivalent to a lattice of 1D wires pointing in the direction of the field and separated by distance $\sqrt{2\pi}\ell_B$, similar to the simplified picture mentioned above[14]. The problem of an array of wires coupled via the long-range Coulomb interaction [Dzyaloshinskii-Larkin (DL) model[10]] can be solved exactly either via summation of diagram series[10] or bosonization[15]. Utilizing the latter technique, we derive the "bulk" tunneling density of states. This quantity also exhibits a power-law anomaly (2) with a different exponent α. Different values of bulk and boundary exponents is another characteristic feature of a LL[11]. We calculate explicitly the values of the exponents α and α_s in the limit of a long-range interaction potential,

$$\kappa^B \ell_B \ll 1, \tag{3}$$

where $\kappa^B = \omega_p / v_F^B$ is the (field-dependent) inverse screening length, and ω_p is the zero-field plasma frequency. Condition (3) is satisfied in magnetic fields $B \lesssim B_0 \equiv \pi^3 \epsilon n / m_{||} e^3$, where ϵ is the background dielectric constant. The introduced here characteristic magnetic field B_0 can be expressed as $B_0 \simeq B_q / r_s$ in terms of the magnetic field B_q de-populating all but the lowest Landau level, and of the gas parameter r_s. Under condition (3), we find

$$\alpha_s = 2\alpha \ln \frac{1}{\sqrt{\alpha}}, \quad \alpha = \frac{e^2}{2\pi\epsilon v_F^B} \equiv \frac{B}{4B_0}. \tag{4}$$

The tunneling current-voltage characteristic is thus a power-law

$$I \propto |V|^{\alpha_s+1}\mathrm{sgn V} \tag{5}$$

for $e|V| \ll \varepsilon_F^B$. Eq. (4) refers to the situation when the magnetic field is perpendicular to the plane of the tunneling contact. If the field is parallel to the contact plane, electrons move along skipping orbits. Interactions between electrons moving in the same direction do not result in anomalous scaling, therefore there is no tunneling anomaly for this field orientation. This means that by tilting the field one can vary the tunneling exponent from its maximum value, given by Eq. (4), to zero.

Another indication of the tunneling anomaly comes from the gapless behavior of a plasmon in a strong magnetic field. For $\omega_c \gg \omega$, where $\omega_c = eB/m_c$ is the cyclotron frequency, the dispersion relation for a classical magnetoplasmon is given by $\omega^2 = \left(k_z^2/k^2\right)\omega_p^2$ [16]. As we see, $\omega \to 0$ for $k_z \to 0$ at finite k. A gapless charge mode slows down the relaxation of an excessive charge added to the conductor in a tunneling event. This mechanism is responsible for the zero-bias anomalies in tunneling into disordered metals and 1D Luttinger liquids. A metal in a strong magnetic field provides one more example of such a behavior.

The LL is destroyed either by backscattering between electrons on different "wires", which results in a CDW-instability, or by formation of bound electron-hole pairs (for the case of a semimetal), which leads to an EI-instability. Also, impurity scattering transfers electrons between the "wires" and thus destroys the 1D motion. The relevant energy scales for these processes is the energy gap, Δ, of either CDW- or EI-origin, and the level broadening, Γ, respectively. On the other hand, a LL-behavior sets in at energies smaller than the Fermi energy ε_F^B. The gap Δ increases with the field, whereas ε_F^B decreases with the field. The LL-state should thus exist in the energy interval which narrows down as B increases:

$$\max\{\Delta(\mathrm{B}), \Gamma\} \lesssim \varepsilon \ll \varepsilon_{\mathrm{F}}^{\mathrm{B}} \propto \mathrm{B}^{-2}. \tag{6}$$

Before giving the derivation of the results announced above, we would like to discuss their relevance to the experiment. The search for LL-like tunneling anomalies in truly 1D systems (edges of 2D electron gas in the FQHE regime[17], carbon nanotubes[18], and quantum wires[19]) has been a very active but remarkably difficult area over the last few years. Compared to experiments on tunneling into 1D systems, tunneling into a 3D system in the ultra-quantum limit has the obvious advantage of the macroscopic system size. It is also advantageous that the tunneling exponent is a function of the external parameters (magnitude and direction of the magnetic field), which can be varied over a wide range.

The choice of the right material is crucial. Among the low-density semimetals (Bi and its alloys and graphite) used conventionally for high-field studies, graphite seems to be the optimal candidate. For B along the c-axis, the ultra-quantum limit is achieved at $B \simeq 7$ T. With material parameters for graphite[20] ($m_{||} = 10m_0$, $\epsilon = 6$, and $n = 3 \times 10^{18}\,\mathrm{cm}^{-3}$), Eq. (4) gives $\alpha = 0.8$ already at 7 T. For comparison, an anomalously large value of the background dielectric constant ($\epsilon \simeq 100$) for Bi[21] and a smaller value of $m_{||}$ ($= 0.5m_0$ for holes) makes α to be very small: even for $B = 100$ T, one has $\alpha = 0.05$. According to a recent experiment[22], the field-induced CDW gap in graphite $\Delta \lesssim 1$ K for $B \lesssim 30$ T and takes its maximum value ($\Delta \simeq 10$ K) at $B \simeq 50$ T. For fields in the range $7\,\mathrm{T} \lesssim B \lesssim 30\,\mathrm{T}$, inequality (6) is satisfied for more than two decades of energies (voltages).

In the limit of weak electron-electron interactions, one can derive the field-induced zero-bias tunneling anomaly (5) by calculating interaction corrections to the transmission coefficient through a tunneling barrier[23]. These corrections can be then summed up by using the renormalization group procedure.

Let a potential barrier of transmission coefficient $t_0 \ll 1$ separate two metallic half-spaces, $z < 0$ and $z > 0$. The magnetic field is perpendicular to the contact plane. We assume that the base-electrode ($z < 0$) is made of a high-density metal and therefore is not affected by the magnetic field. Electron-electron interactions are then not important in this half-space and we treat it as a Fermi gas. The counter-electrode ($z > 0$) is a low-density metal in the ultra-quantum magnetic limit. In this half-space, electrons interact via potential $U(\mathbf{r} - \mathbf{r}')$. Keeping only the leading terms in $t_0 \ll 1$, the free wave-function in the Landau gauge $\mathbf{A} = (-yB, 0, 0)$ for $z > 0$ and far away from the barrier is given by

$$\Psi^0_{p_x,-p_z} = -\left(2i \sin p_z z/\sqrt{L_z}\right) \Phi^0_{p_x}(\mathbf{r}_\perp) \tag{7a}$$

$$\Psi^0_{p_x,p_z} = t_0 \left(e^{ip_z z}/\sqrt{L_z}\right) \Phi^0_{p_x}(\mathbf{r}_\perp), \tag{7b}$$

where $p_z = \sqrt{2m\varepsilon} > 0$, $\mathbf{r}_\perp \equiv (x, y)$ and

$$\Phi^0_{p_x}(\mathbf{r}_\perp) = (\sqrt{\pi}\ell_B L_x)^{-1/2} e^{ip_x x} \exp\left[-(y + p_x\ell_B^2)^2/2\ell_B^2\right].$$

The leading correction to the transmission coefficient comes from the exchange interaction[23], if the interaction potential is smooth on the scale of Fermi wavelength, see Eq. (3).

The correction to the wave caused by the exchange potential V_x has the form

$$\Psi_{p_x,p_z}(\mathbf{r}) = \Psi^0_{p_x,p_z}(\mathbf{r}) + \int\int d\mathbf{r}' d\mathbf{r}'' G^{>}_{\varepsilon_z}(\mathbf{r}, \mathbf{r}') \times V_x(\mathbf{r}', \mathbf{r}'') \Psi^0_{p_x,p_z}(\mathbf{r}''), \tag{8}$$

where $G^{>}_{\varepsilon_z}(\mathbf{r}, \mathbf{r}')$ is the Green's function of the free Schrödinger equation in the half-space $z > 0$, and $\varepsilon_z \equiv p_z/2m^2$ is the energy of electron motion along the field. The integration in (8) goes only over those regions where electrons interact, i.e., $z', z'' \geq 0$.

The renormalized transmission coefficient is extracted from the asymptotic form of Ψ_{p_x,p_z} for $z \to +\infty$ which, in its turn, is determined by the asymptotic behavior of $G^{>}_{\varepsilon_z}(\mathbf{r}, \mathbf{r}')$ [see Eq. (8)]. The asymptotic form of

the Green's function in the infinite space can be written as $G_{p_z}(\mathbf{r},\mathbf{r}')|_{z\to+\infty}=G_{||}(z,z',p_z)G_\perp(\mathbf{r}_\perp,\mathbf{r}'_\perp)$, where $G^{||}_{\varepsilon_z}(z,z')=(m_{||}/ip_z)e^{ip_z|z-z'|}$ is the Green's function for the 1D motion along the z-axis, and

$$G_\perp(\mathbf{r}_\perp,\mathbf{r}'_\perp)=\frac{1}{2\pi\ell_B^2}e^{-\left[\mathbf{r}_\perp-\mathbf{r}'_\perp\right]^2/4\ell_B^2}e^{-i(y+y')(x-x')/2\ell_B^2}. \tag{9}$$

The asymptotic form of $G^{>}_{\varepsilon_z}(\mathbf{r},\mathbf{r}')$ is then obtained via the method of images

$$G^{>}_{\varepsilon_z}(\mathbf{r},\mathbf{r}')|_{z\to+\infty}=\left[G^{||}_{p_z}(z,z')-G^{||}_{p_z}(z,-z')\right]G_\perp(\mathbf{r}_\perp,\mathbf{r}'_\perp).$$

The exchange potential is given by

$$\begin{aligned}V_x(\mathbf{r},\mathbf{r}')&=-U(\mathbf{r}-\mathbf{r}')\int_0^{p_F^B}\frac{dp_z}{2\pi}\int_{-\infty}^{\infty}\frac{dp_x}{2\pi}\left[\Psi^0_{p_x,-p_z}(\mathbf{r}')\right]^*\\&\quad\times\Psi^0_{p_x,-p_z}(\mathbf{r})\\&\approx-U(\mathbf{r}-\mathbf{r}')\frac{\sin p_F^B(z+z')}{\pi(z+z')}G_\perp(\mathbf{r}_\perp,\mathbf{r}'_\perp),\end{aligned} \tag{10}$$

where $p_F^B=m_{||}v_F^B$. In Eq. (10), we retained only that term which at $p_z\approx p_F^B$ leads to a logarithmic divergence in the integral over $z+z'$ in Eq. (8).

The transmission coefficient in the first order with respect to interactions is

$$t=t_0\left(1-\frac{e^2}{2\pi\epsilon v_F^B}\ln\frac{1}{\kappa^B\ell_B}\ln\frac{1}{\ell_B|p_z-p_F^B|}\right), \tag{11}$$

where the inverse screening radius

$$\kappa^B=\omega_p/v_F^B=2\alpha{\ell_B}^{-1}\propto B. \tag{12}$$

This result is valid if both logarithmic factors are large. The contribution t from the Hartree term is smaller than (11) by $\ln(1/\kappa^B\ell_B)$. The higher order terms in the expansion of t can be summed up via the renormalization group procedure[23]. The result is

$$t\propto|p_z-p_F^B|^{\alpha_s/2}\propto|\varepsilon-\varepsilon_F^B|^{\alpha_s/2}. \tag{13}$$

As $I\propto|t|^2V$, Eq. (13) yields Eq. (5) with exponent (4). The perturbative result (13) is valid for $\alpha\ll1$, which means that α_s is also small within this approach.

We now demonstrate how the LL-state emerges in the coherent state formulation. The method of bosonization, applied in this formulation, provides a simple tool for calculation the "bulk" 1D density of states[11]. Also, it allows us to estimate the tunneling exponents for stronger interaction ($\alpha\gtrsim1$). The Ψ-operator is expanded over the coherent states basis

$$\Psi(\mathbf{r})=\sum_N\sum_{\mathbf{R}}a^\dagger_{N\mathbf{R}}(z)\chi_{N\mathbf{R}}(\mathbf{r}_\perp), \tag{14}$$

where $\mathbf{r}$=$(z,\mathbf{r}_\perp)$, $\mathbf{R}$=(R_x,R_y). A coherent state $\chi_{N\mathbf{R}}(\mathbf{r}_\perp)$ is a simultaneous eigenstate of the energy operator

$$(2m_c)^{-1}\left(\mathbf{p}_\perp-e\mathbf{A}\right)^2\chi_{N\mathbf{R}}=\omega_c(N+1/2)\chi_{N\mathbf{R}},$$

and of the "guiding center" operator

$$(\hat{x}_0-i\,\hat{y}_0)\chi_{N\mathbf{R}}=(R_x-iR_y)\chi_{N\mathbf{R}},$$

where $\hat{x}_0=\hat{x}+(m\omega_c)^{-1}(\hat{p}_y-eA_y)$ and $\hat{y}_0=\hat{y}-(m\omega_c)^{-1}(\hat{p}_x-eA_x)$ are the operators corresponding to the classical coordinates of guiding centers[24]. In the symmetric gauge, $\mathbf{A}=(1/2)\mathbf{B}\times\mathbf{r}$, an explicit form of the CS corresponding to $N=0$ is[24]

$$\chi_{0\mathbf{R}}(\mathbf{r}_\perp)=\frac{1}{\sqrt{2\pi}\ell_B}\exp\left[-\frac{\left(\mathbf{r}_\perp-\mathbf{R}\right)^2+2i\mathbf{r}_\perp\wedge\mathbf{R}}{4\ell_B^2}\right],$$

where $\mathbf{a}\wedge\mathbf{b}=a_xb_y-a_yb_x$.

As any set of coherent states, the set of $\chi_{N\mathbf{R}}$ remains (over) complete even if defined on a subset of points in the complex plane, R_x+iR_y, rather than on the whole plane. The von Neumann theorem[25] allows one to choose this subset as sites of a 2D square lattice. To fix the value of the lattice spacing, one notices that a total of n particles per unit volume are now distributed over "wires", which are parallel to the magnetic field and cross the plane perpendicular to the field at the sites of the von Neumann lattice. For $N=0$, the number density per unit length of each "wire" is p_F^B/π, their the areal density is $1/s^2$, thus $n=p_F^B/\pi s^2$. Comparing this equation to (1), one gets $s=\sqrt{2\pi}\ell_B$. This is a maximum value of the lattice spacing for which the set is still complete. States of the CS basis on the von Neumann lattice describe particles which are localized on the lattice sites in the direction transverse to the field but free to move along the field. This picture is in a close resemblance to the classical trajectories spiraling around the field lines.

Coherent states are not orthogonal. In particular,

$$\langle\chi_{0\mathbf{R}}|\chi_{0\mathbf{R}'}\rangle=\frac{1}{2}\exp\left[-\frac{\left(\mathbf{R}-\mathbf{R}'\right)^2+2i\mathbf{R}\wedge\mathbf{R}'}{4\ell_B^2}\right], \tag{15}$$

which can be replaced by $2\pi\ell_B^2\delta^2(\mathbf{R}-\mathbf{R}')$ in the limit $\ell_B\to0$. Only in this limit the coherent set diagonalizes the quadratic part of the Hamiltonian. Physically, this limit means that ℓ_B is much less than the characteristic value of $|\mathbf{R}-\mathbf{R}'|$. For interacting electrons, typical $|\mathbf{R}-\mathbf{R}'|$ is of the order of the screening radius $1/\kappa^B$. Thus the approach based on coherent states is valid for a long-range interaction as defined by condition (3).

Using asymptotic orthogonality of the coherent states, one arrives at the effective 1D Hamiltonian (from now on we concentrate on $N=0$ and suppress index N)

$$\begin{aligned}H&=\sum_{\mathbf{R}}\int dz a^\dagger_{\mathbf{R}}(z)\left[\frac{\omega_c}{2}+\frac{p_z^2}{2m_{||}}\right]a_{\mathbf{R}}(z)\\&\quad+\frac{e^2}{2\epsilon}\sum_{\mathbf{R},\mathbf{R}'}\int dzdz'\frac{a^\dagger_{\mathbf{R}}(z)a^\dagger_{\mathbf{R}'}(z')\hat{a}_{\mathbf{R}''}(z')a_{\mathbf{R}}(z)}{\sqrt{\left(\mathbf{R}-\mathbf{R}'\right)^2+\left(z-z'\right)^2}}.\end{aligned} \tag{16}$$

Due to the long-range nature of the Coulomb potential, it suffices to keep only the forward scattering processes in (16) (both for $\mathbf{R}=\mathbf{R}'$ and $\mathbf{R}\neq\mathbf{R}'$), in which case (16) is identical to the DL model[10]. Backscattering for $\mathbf{R}\neq\mathbf{R}'$ leads to a CDW instability[2,13]. The forward scattering part of (16) is diagonalized via bosonizing the fermions

$$a_{\mathbf{R}} =: e^{-i\left(p_F^B z+\sqrt{\pi}\{\phi_{\mathbf{R}}+\theta_{\mathbf{R}}\}\right)} + e^{i\left(p_F^B z+\sqrt{\pi}\{\phi_{\mathbf{R}}-\theta_{\mathbf{R}}\}\right)} :,$$

where $[\phi_{\mathbf{R}}(z_1),\partial_{z_2}\theta_{\mathbf{R}'}(z_2)] = i\delta_{\mathbf{R},\mathbf{R}'}\delta(z_1-z_2)$. The result of a straightforward calculation for the equal-point (Matsubara) Green's function is

$$\mathcal{G}(\tau)\sim\tau^{-1}e^{-f(\tau)},$$
$$f(\tau)=2\pi^2\ell_B^2\int_{\mathrm{BZ}}\frac{d^2q_\perp}{(2\pi)^2}\int_0^\Lambda\frac{dq_z}{2\pi}\frac{(u-1)^2}{u}\frac{1-e^{-uv_F^B q_z\tau}}{q_z},$$
$$u^2\equiv 1+(\kappa^B)^2\sum_{\mathbf{G}}(\mathbf{Q}+\mathbf{G})^{-2},\ \mathbf{Q}=(\mathbf{q}_\perp,q_z).$$

Here the $q_\perp$-integration goes over the Brillouin zone (BZ) of the von Neumann lattice, $\mathbf{G}$ are the reciprocal lattice vectors, Λ is the cutoff, and uv_F^B has the meaning of **a** plasmon velocity. Typical q_z in $f(\tau)$ are determined by the low-energy scale of the problem ($\max\{\mathrm{T},\mathrm{eV}\}$) and thus small, which allows one to neglect q_z in function u. The DOS reduces to the form (2) with an exponent

$$\alpha=\pi\ell_B^2\int_{\mathrm{BZ}}\frac{d^2q_\perp}{(2\pi)^2}\frac{(u-1)^2}{u}. \qquad (18)$$

For $\kappa^B\ell_B\ll 1$, the sum over the reciprocal vectors in (18) is dominated by the $\mathbf{G}=0$ term so that $u^2\approx 1+(\kappa^B/q_\perp)^2$. The main contribution to the integral over $q_\perp$ in (18) comes from the region $q_\perp\simeq\kappa^B\ll 1/\ell_B\simeq G$, thus the integration can be extended over the entire q-space. The bulk DOS behaves as $|\varepsilon-\varepsilon_F^B|^\alpha$, where α is given in Eq. (4).

Note that in the main contribution to α [see Eq. (18)], a typical value of $u-1$ is of the order of unity. This indicates a non-perturbative nature of the result for α. The lowest-order perturbation theory, applied to the bulk case, would have given $\alpha\propto e^4$, whereas the correct result is $\alpha\propto e^2$.

Tunneling into the end of a semi-infinite sample can be treated by imposing the boundary condition on the current operator: $j_{\mathbf{R}}=(1/i\sqrt{\pi})\partial_\tau\phi_{\mathbf{R}}=(1/\sqrt{\pi})\partial_x\theta_{\mathbf{R}}=0$ for $z=0$. This translates into the boundary conditions for the propagators: $\mathcal{P}^\rho_{\mathbf{R}}(z,z',\tau)=\langle\rho_{\mathbf{R}}(z,\tau)\rho_{\mathbf{R}}(z',0)-\rho^2_{\mathbf{R}}(z,0)\rangle$, where $\rho=\phi,\theta$. $\mathcal{P}^\rho_{\mathbf{R}}$ can be constructed from the propagators for the infinite medium, $P^\rho_{\mathbf{R}}$, by using the method of images. Evidently, $\mathcal{P}^\phi_{\mathbf{R}}=0$ at the boundary whereas $\mathcal{P}^\theta_{\mathbf{R}}(0,0,\tau)=\pi^{-1}\int dq_z P^\theta_{\mathbf{R}}(q_z,\tau)$. The DOS at the boundary is again of form (2) but with an exponent

$$\alpha_s=2\pi\ell_B^2\int_{\mathrm{BZ}}\frac{d^2q_\perp}{(2\pi)^2}(u-1). \qquad (19)$$

In contrast to the bulk case, the logarithmically divergent integral over $q_\perp$ in (19) is cut off at the BZ boundary, *i.e.*, for $q_\perp\simeq 1/\ell_B$. Therefore, the condition of asymptotic orthogonality of CS is satisfied only with the logarithmic accuracy. The result for α_s is given by Eq. (4).

In the strong-coupling case ($\alpha\gtrsim 1$), typical $q_\perp\simeq 1/\ell_B$, which means that the approach based on CS becomes invalid quantitatively. However, one can still estimate tunneling exponents (both for the "bulk" and "edge tunneling) as $\tilde{\alpha}=C\sqrt{e^2/v_F^B}$, where the numerical constant $C\simeq 1$ cannot be determined by this method.

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